Project Title
COSMOLOGIES WITH DARK MATTER, DARK ENERGY & THE MODIFIED GRAVITY
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Abstract

In the project cosmological models have been investigated taking into account modified gravitational sector and/or modified matter sector. The standard BigBang model was the beginning to understand the observed universe immediately after it was known that the universe is expanding. Hubble's discovery made it clear that we are not living in a static Einstein's universe. Afterwards it is found that the although BigBang model satisfactorily explains some of the observed facts of the universe it is entangled with few conceptual problems which finds no answer in the framework of perfect fluid model. Then it was claimed that the there exists an inflationary phase of expansion of the universe which we need it because of marriage of the particle physics with cosmology. In 1991, the theoretical cosmological models came up to describe the evolution of the universe may be right from the Planck time. The existence of such an early inflation is also predicted by Cosmic Background Explorer (COBE). In this context a modification of matter sector *i.e.,* introduction of scalar field available in standard model of particle physics, instead of isotropic fluid and/or a modification of gravitational sector by Starobinsky came up to obtain early inflation. In 1999, a new phenomena is observed which predict that the universe is passing through an accelerating phase. In general theory of relativity (GTR), it came up that once again we need a further modification of the matter or gravitational sector. Consequently new physics and new concepts are needed to understand the observed universe. In view of the recent astronomical and cosmological observations the idea of dark energy and dark matter is conceived in theoretical framework. In the project the recent issues in cosmology are probed considering modified gravity or GTR with modified matter sector. It came up that although one can construct a number of cosmological models, some of them are ruled out when analyzed with observations. Recent observed data from astronomical and cosmological observations have

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revealed some novel features of the universe imposing constraints on cosmological models. A number of observed data in future cosmological missions will help further to understand the origin of the universe in a better way.

1 Introduction

In modern cosmology, inflation [1, 2, 3, 4, 10, 9, 11, 12] in the early universe is one of the essential ingredient for constructing a viable cosmological model. Existence of such a phase of inflationary expansion in the early universe is supported from observations of Cosmic Microwave Background Radiation explorer (COBE). The overlapping region of particle physics and cosmology help us to understood the early universe in a better way. The success of inflationary models [5, 6, 7], in particular, the predicted primordial density fluctuations for the structure formation of the universe is in agreement with the COBE data. In addition it opens up new avenues in the interface of particle physics and cosmology leading to new insights both in conceptual and technical issues. The theory of inflation is attractive as it addressed some of the longstanding problems not understood in the perfect fluid model satisfactorily. A volume of literature with various cosmological models appeared in the last 30 years to understand the origin of the universe. Although it has a number of good features, its mechanism of realization still remains *ad hoc*.

On the otherhand, the recent observed astronomical and cosmological [16, 17, 15, 18, 19] data when interpreted in the context of the Big bang model have provided some interesting information on the composition of fluids in the universe. The prediction for the composition of the universe namely, dark energy and dark matter in addition to normal matter come from Supernova redshift survey, Wilkinson Microwave Anisotropy Probe (WMAP), velocity rotational curves in galaxies, the gravitational lensing of galactic clusters [1]. The total energy density may be made up of three components, baryonic matter 4 %, dark matter 23 % and third part, called dark energy which constitutes the rest 73 % an entirely new kind of fluid not known in the framework of standard model of particle physics. The present universe is passing once again through an accelerating phase of the universe. As discussed the present universe might have emerged from an inflationary era in the very early phase of the evolution and thereafter the universe passes through radiation dominated and matter dominated phases. Subsequently recent predictions that the present universe is passing through an accelerating phase. It is not possible to accommodate all these phases in a single theoretical framework. Moreover, the general theory of relativity which is the key theory to understand the universe and its evolution is not enough for understanding the observed universe. This is a challenge in theoretical physics as fields known in the standard model of particle physics are also not enough for understanding the present universe although scalar fields can describe the evolution of the early universe. It is, therefore, demanded that either the gravity sector or the matter sector requires to be modified. The proper cause of recent acceleration is yet to be understood. It is, therefore, realized that the Einstein field equation with matter fields permitted by the standard model of particle physics is not enough to accommodate the accelerating phase. Thus describing universe with present acceleration is a challenging job for theoretical physics. A number of literature came up with cosmological model making use of dark energy and dark matter to accommodate present accelerating phase but source of the new form of matter remains unknown. We do not know why the universe is still accelerating, but answer to the question definitely will require a new fundamental physics not known yet. In this context we intend to investigate alternative theory considering a modified sector of gravitational and matter sector of the Einstein gravitational action and confront the results obtained with the observational data. It may be mentioned here that Starobinsky [2] first obtained inflation in cosmology adding a R^2 term in the Einstein-Hilbert action long before the efficacy of inflation is known. In the same way a modified gravity with curvature terms relevant for the present epoch of the universe may be important to describe the present accelerating phase. Thus cosmological models with modified gravity or in the presence of dark matter and dark energy are important to look for a universe that accommodates the present accelerating phase.

1.1 Objective :

The objective of the project is to explore cosmological model which can accommodate the present accelerating universe in addition to the usual phases of the early universe. The nature of dark matter and dark energy is to be explored considering modified theories of gravity and/or modified matter sector of gravity.

1.2 Methodology :

The Einstein's general theory of relativity (GTR) is the key equation in understanding the evolution of the universe. As the recent observations are not fully understood in the framework of Einstein-Hilbert action. A modified gravitational sector which may be a correction of GTR and/or a modified matter sector will be employed here. As the fields in the standard model confronts with observations in the context of GTR cosmology, new fields possibly new physics from Large Hadron Collider (LHC) may be needed. In this context a polynomial function of Ricci scalar for gravitational sector and exotic fields in the case of matter sector are usually prescribed.

In Big bang model a number of issues namely, singularity, existence of quantum gravity region also plagued the model. A model of an ever existing universe (singularity free) which eventually enters into the standard big bang epoch at some stage and consistent with the features known today is constructed in [27] which we call emergent universe (EU) model. A flat EU model can be implemented with a composition of normal and exotic matter permitted by the non-linear equation of state (EOS) $p = A\rho - B\sqrt{\rho}$, (A and B are arbitrary constants). The results expected from PLANCK will be helpful in understanding the EU model in a better way in addition to other cosmological models in various modified theories of gravity. To obtain a viable cosmological model we use observational results and analyze with *chi*-square minimization technique [5]. In the case of non-linear EOS the field equations become very much complex to obtain a cosmological solution in known functional form. Therefore, it is convenient to adopt numerical techniques and consequently using Mathematica and MATLAB, cosmological models have been numerical analysis with the observed data.

The gravitational action in (3+1) dimensions is given by

$$I = \frac{1}{16\pi G} \int R\sqrt{-g} d^4x + I_m \tag{1}$$

where G is Newton's gravitational constant, R is the Ricci scalar, g is the determinant of the metric and I_m is the matter action. The Einstein field equation is obtained by varying the above action which is given as

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu} \tag{2}$$

where $R_{\mu\nu}$, R, $g_{\mu\nu}$, $T_{\mu\nu}$ and G represent the Ricci tensor, Ricci scalar, metric tensor, matter-energy tensor and Newton's gravitational constant. Here we consider four dimensions for which μ, ν runs from (0, 1, 2, 3).

We consider a flat Robertson-Walker metric which is given by

$$ds^2 = -dt^2 + a^2(t)\left[dr^2 + r^2\left(d\theta^2 + sin^2\theta d\phi^2\right)\right] \tag{3}$$

where $a(t)$ represents the scale factor of the Universe. We consider the energy momentum tensor as $T^{\mu}_{\nu} = diagonal(\rho, -p, -p, -p)$, where ρ is the energy density and p is the pressure. Using the flat Robertson-Walker metric given by eq. (3) in the Einstein's field equation one obtains

$$\rho = 3\left(\frac{\dot{a}}{a}\right)^2, \tag{4}$$

$$p = -\left[2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right] \tag{5}$$

where we set $G = \frac{1}{8\pi}$ and $c = 1$. The conservation equation is given by

$$\frac{d\rho}{dt} + 3H\left(p + \rho\right) = 0 \tag{6}$$

where $H = \frac{\dot{a}}{a}$ represents the Hubble parameter.

1.3 Results Expected :

Constraints imposed from observational data will hopefully help us to eliminate some of the possible models and facilitate in recognition of probable satisfactory model.

1st Year : Since solution of EU model is already been obtained earlier we will compare and confront the predictions of this model and F(R) models with the observational data.

2nd Year : We will set up the field equation for a universe with cosmological constant and various matter fields and look for its solution. The experience of the work done in the first year will help us to constrain the choice of matter fields.

3rd year : The complex evolution of the universe, the peculiar change over in the expansion of the universe will be studied in the context of cosmological models considered earlier.

2 Problems Investigated

The following works are carried out

I. Cosmology

- Emergent Universe Model
- Evolution of Primordial Black hole
- Observational constraints on modified Chaplygin gas

II Astrophysics

- Compact Objects

3 Cosmology I : Emergent Universe Model

In spite of its overwhelming success, modern big-bang cosmology still has some unresolved issues. The physics of the inflation [1, 9, 10, 11, 12, 13, 14] and the introduction of a small cosmological constant for late time acceleration [15, 16, 17] are not completely understood [20, 21] in details. Moreover, various competing models exist that are as yet not fully distinguished empirically from the currently available observational data. For this reason there is enough motivation to search for an alternative cosmological model. In this context, Ellis and Maartens [22] considered the possibility of a cosmological model [23] in which there is no big-bang singularity, no beginning of time, and the universe effectively get rid of a quantum regime for space-time by staying large at all times. The universe started out in the infinite past in an almost static Einstein universe, and subsequently, it entered in an expanding phase slowly, eventually evolving into a hot big-bang era.

The EU scenario merits attention, as it promises to solve several conceptual as well as technical issues of the big-bang model. A notable direction is regarding the cosmological constant problem [26]. Mukherjee *et al.* [27] obtained an emergent universe in the framework of general theory of relativity in a flat universe with a nonlinear equation of state of the form

$$p = A\rho - B\sqrt{\rho} \tag{7}$$

where A and B are arbitrary constants. Emergent universe accommodates a late time de-Sitter phase and thus it naturally leads to the late time acceleration of the universe, as well. Such a scenario is promising from the perspective of offering unified early as well as late time dynamics of the universe [39]. Note, however, that the focal point of unification in such emergent universe models lies in the choice of the equation of state for the polytropic fluid, while several other models of unification rely more on the scalar field dynamics through choice of field potentials [40, 41, 42].

The emergent model proposed by Mukherjee *et al.* [27] gives rise to a universe with a composition of three different types of fluids determined by the parameters A and B. In the original emergent universe model proposed by Mukherjee *et al.* [27], it was assumed non-interacting fluids and each of the three types of fluids identified satisfy conservation equations separately. For a viable cosmological scenario, it is further necessary to consider a consistent model of the universe which contains radiation dominated, matter dominated and subsequently the late accelerated phases of the universe. In the original EU model [27], the composition of the universe is fixed once A is fixed. A problem thus arises as to how a pressureless matter component could be accommodated within such a scenario. However, allowing interaction among the constituent fluids of the emergent universe may open up richer physical consequences. In the present context interaction among the constituent fluids is useful to obtain a consistent evolutionary scenario of the universe. Another important consistency condition is imposed through the thermodynamics of an expanding universe. Einstein's equations have been interpreted as a thermodynamical relation resulting from the displacement of the horizon. Since the emergent universe scenario entails a phase of accelerated expansion, it is relevant here to study the status of the second law of thermodynamics in the picture involving interacting fluids in the emergent universe.

Two different types of emergent universe scenario are studied: (i) a two-fluid model with interaction of the fluid having the non-linear EoS given by eq. (1) with another barotropic fluid, beginning at some time $t = t_i$ (Model-i) and (ii) a three-fluid model with interaction among the various constituent fluids with different individual equations of state, starting at a time $t = t_o$ (Model-ii).

In the original emergent universe [27] a polytropic equation of state (henceforth, EoS) is considered which is

$$p = A\rho - B\rho^{1/2} \tag{8}$$

where A and B are arbitrary constants. Making use of the conservation equation and the EoS given by eqs. (4) and (5) in eq. (7), one obtains a second order differential equation given by

$$2\frac{\ddot{a}}{a} + (3A+1)\left(\frac{\dot{a}}{a}\right)^2 - \sqrt{3}B\frac{\dot{a}}{a} = 0. \tag{9}$$

The scale factor of the universe is thus obtained integrating eq. (8) which is given by

$$a(t) = \left[\frac{3K(A+1)}{2}\left(\sigma + \frac{2}{\sqrt{3}B}\, e^{\frac{\sqrt{3}}{2}Bt}\right)\right]^{\frac{2}{3(A+1)}} \tag{10}$$

where K and σ are the two integration constants. It is interesting to note that $B < 0$ leads to a contracting universe whereas with $B > 0$ and $A > -1$ leads to a non-singular solution which is expanding. The later solution corresponds to an emergent universe which was obtained by Mukherjee *et. al.* [27]. The energy density of the universe in terms of scale factor is obtained from eq. (6) making use of EoS given by eq. (7) which is given by

$$\rho(a) = \frac{1}{(A+1)^2}\left(B + \frac{K}{a^{\frac{3(A+1)}{2}}}\right)^2. \tag{11}$$

Expanding the above expression, one obtains energy density as the sum of three terms which can be identified with three different types of fluids. Thus, the components of energy density and pressure can be expressed as follows:

$$\rho(a) = \Sigma_{i=1}^{3}\rho_i \quad and \quad p(a) = \Sigma_{i=1}^{3}p_i \tag{12}$$

where we denote

$$\rho_1 = \frac{B^2}{(A+1)^2}, \quad \rho_2 = \frac{2KB}{(A+1)^2}\frac{1}{a^{\frac{3(A+1)}{2}}}, \quad \rho_3 = \frac{K^2}{(A+1)^2}\frac{1}{a^{3(A+1)}} \tag{13}$$

$$p_1 == -\frac{B^2}{(A+1)^2}, \quad p_2 = \frac{KB(A-1)}{(A+1)^2}\frac{1}{a^{\frac{3(A+1)}{2}}}, \quad p_3 = \frac{AK^2}{(A+1)^2}\frac{1}{a^{3(A+1)}}. \tag{14}$$

Comparing with the barotropic EoS , one can determine the role of A. For example, $A = \frac{1}{3}$ leads to a universe with radiation, exotic matter and dark energy, $A = 0$ leads to dark energy, exotic matter and dust. Thus once the EoS parameter A is fixed the composition of the fluid in the universe gets determined. It is not possible to get a universe at a later epoch with matter in it. We consider the following two models :

Model (i) : The two fluids model

We consider two interacting fluids with densities ρ and ρ' which can exchange energies with each other. One of the fluid with energy density, say ρ is dominated to begin with satisfying a non-linear EoS given by eq. (1) which leads to an emergent universe model. The effect of other fluid in the energy density of the universe is assumed to be important at a later epoch. The pressure of the former fluid is

$$p = A\rho - B\rho^{1/2}. \tag{15}$$

where A and B are constants. The other fluid is considered to be barotropic which is given by

$$p' = \omega' \rho' \tag{16}$$

where ω' is EoS parameter. The Hubble parameter $(\left(H = \frac{\dot{a}}{a}\right))$ is given by

$$3H^2 = \rho + \rho'. \tag{17}$$

In this case exchange of energy between two different fluids is allowed. The interaction may sets at t_i. The two interacting fluids respect a total energy conservation equation and their densities evolve with time as

$$\dot{\rho} + 3H(\rho + p) = -\alpha\rho H, \tag{18}$$

$$\dot{\rho'} + 3H(\rho' + p') = \alpha\rho H \tag{19}$$

where α represents a coupling parametrizing the energy exchange between the fluids. One may view the above interaction as a flow of energy from first kind of fluid to the second one (say, dark matter) beginning at the epoch considered here. Now, using eq. (14) in eq. (17), we get a first order differential equation which can be integrated to obtain the behaviour of energy density in terms of the scale factor of the universe and the interaction coupling factor. Thus the energy density and pressure for the fluid of the first kind are given by

$$\rho = \frac{B^2}{(A+1+\frac{\alpha}{3})^2} + \frac{2KB}{(A+1+\frac{\alpha}{3})^2}\frac{1}{a^{\frac{3(A+1+\frac{\alpha}{3})}{2}}} + \frac{K^2}{(A+1+\frac{\alpha}{3})^2}\frac{1}{a^{3(A+1+\frac{\alpha}{3})}}, \tag{20}$$

$$p = -\frac{B^2}{(A+1+\frac{\alpha}{3})^2} + \frac{KB(A-1+\frac{\alpha}{3})}{(A+1+\frac{\alpha}{3})^2}\frac{1}{a^{\frac{3(A+1+\frac{\alpha}{3})}{2}}} + \frac{(A+\frac{\alpha}{3})K^2}{(A+1+\frac{\alpha}{3})^2}\frac{1}{a^{3(A+1+\frac{\alpha}{3})}}. \tag{21}$$

If the interaction is with a pressureless dark fluid *i.e.*, $p' = 0$, eq. (18) can be integrated using eqs. (16) and (19) which determine the total energy density and pressure as follows :

$$\rho_{total} = \rho + \rho' = \frac{B^2}{A+1} + \frac{2KB}{(A+1)^2}\frac{1}{a^{\frac{3(A+1)}{2}}} + \frac{K^2}{A+1}\frac{1}{a^{3(A+1)}}, \tag{22}$$

$$p_{total} = p = -\frac{B^2}{A+1} + \frac{KB(A-1)}{(A+1)^2}\frac{1}{a^{\frac{3(A+1)}{2}}} + \frac{AK^2}{(A+1)^2}\frac{1}{a^{3(A+1)}}. \tag{23}$$

The equation of state parameter for the second fluid is given by

$$\omega' = \frac{p'}{\rho'} = \frac{p_{total} - p}{\rho_{total} - \rho}. \tag{24}$$

An interesting case emerges when the coupling parameter $\alpha = 2$ and $A = \frac{1}{3}$. In this case a universe with dark energy, exotic matter and radiation to begin with (i.e., before the interaction sets in) can transform to a matter dominated phase. A consistent scenario of the observed universe in the EU model may be realized in this case.

Model (ii) : The three fluids model

The original EU model was obtained in the presence of non-interacting fluids permitted by the parameter A in a flat universe case. The corresponding densities and pressures are given by eqs. (13) and (14) respectively. For non-interacting fluids, the EoS parameters for the three fluids permitted above are given by $\omega_1 = -1, \ \ \omega_2 = \frac{1}{2}(A-1), \ \ \omega_3 = A$. For $0 \leq A \leq 1$, it accommodates dark energy, exotic matter and the usual barotropic fluid. The energy density and pressure of the exotic matter and that of the barotropic fluids decreases with the expansion of the universe. However, the rate of decrease is different evident from eqs. (13) and (14). We assume an interaction among the components of the fluid in the universe which is assumed to be originated at a later epoch (such interactions could arise due to a variety of mechanisms [48, 49, 50, 51, 52, 53]). Assuming onset of interaction among the composition of the fluid at $t \geq t_o$, the conservation equations for the energy densities of the fluids now can be written as

$$\dot{\rho_1} + 3H(\rho_1 + p_1) = -Q', \tag{25}$$

$$\dot{\rho_2} + 3H(\rho_2 + p_2) = Q, \tag{26}$$

$$\dot{\rho_3} + 3H(\rho_3 + p_3) = Q' - Q, \tag{27}$$

where Q and Q' represent the interaction terms, which can have arbitrary form, ρ_1 represents dark energy density, ρ_2 represents exotic matter and ρ_3 represents normal matter. In this case $Q < 0$ corresponds to energy transfer from exotic matter sector to two other constituents, $Q' > 0$ corresponds to energy transfer from dark energy sector to the other two fluids, and $Q' < Q$ corresponds to energy loss for the normal matter sector. The case $Q = Q'$ corresponds to the limiting case where dark energy interacts only with the exotic matter. It is important to see that although the three equations are different the total energy of the fluid satisfies the conservation equation together. It is possible to construct the equivalent effective uncoupled model, described by the following conservation equations:

$$\dot{\rho_1} + 3H(1 + \omega_1^{eff})\rho_1 = 0 \tag{28}$$

$$\dot{\rho_2} + 3H(1 + \omega_2^{eff})\rho_2 = 0 \tag{29}$$

$$\dot{\rho_3} + 3H(1 + \omega_3^{eff})\rho_3 = 0 \tag{30}$$

where the effective equation of state parameters are given below:

$$\omega_1^{eff} = \omega_1 + \frac{Q'}{3H\rho_1}, \tag{31}$$

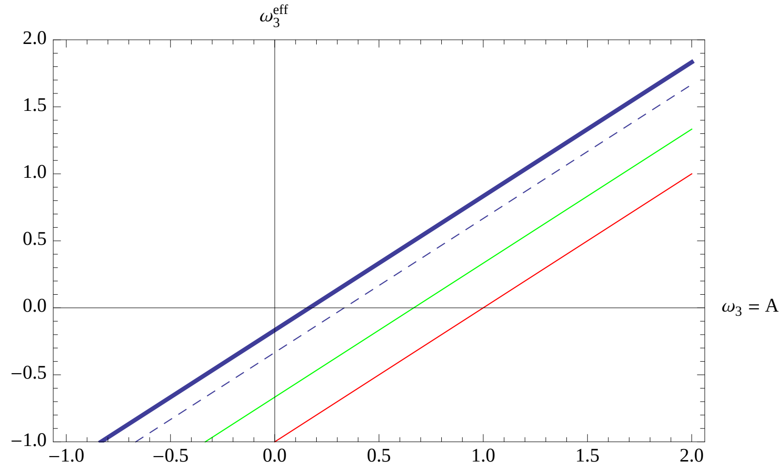

Figure 1: Plot of ω_3^{eff} with EoS parameter A for different interaction β. The thick, dash, red line and green lines are for $\beta = 0.5,\ 1,\ 2,\ 3$ respectively.

$$\omega_2^{eff} = \omega_2 - \frac{Q'}{3H\rho_2}, \tag{32}$$

$$\omega_3^{eff} = \omega_3 + \frac{Q - Q'}{3H\rho_3}. \tag{33}$$

Now, if we consider the interaction as $Q - Q' = -\beta H \rho_3$, the effective state parameter for the normal fluid becomes

$$\omega_3^{eff} = \omega_3 - \frac{\beta}{3} \tag{34}$$

In fig. 1 we plot the variation of effective equation of state parameter ω_3^{eff} with ω_3 (which corresponds to A of the EoS parameter) for different strengths of interaction determined by β. We note that as the strength of interaction is increased the value of ω_3 ($i.e.$, A) for which $\omega^{eff} = 0$ (corresponds to matter domination) is found to increase. Thus a universe with any A value is found to admit a matter dominated phase at a late epoch depending on the strength of the interaction which was not permitted in the absence of interaction in an EU model proposed by Mukherjee *et al.* [27]. It may be pointed out here that in the very early era a universe is assumed with a composition of three different fluids having no interaction in this picture, thus the behaviour of the universe at early times remains unchanged as was found in the original EU model. Thus the emergent universe scenario proposed by Mukherjee *et al.* [27] can be realized in the early era but at a later epoch the composition of matter changes in the present scenario from its original one with the onset of interaction. This feature represents a clear improvement over the earlier cosmological scenario in an emergent universe [27, 29, 30] where it is rather difficult to accommodate a pressureless fluid.

3.1 Concluding Remarks on Emergent Universe Model

Emergent universe scenario is explored in the presence of interacting fluids. Two different cosmological models are Model (i), we consider the flow of energy from the fluids required to realize the emergent universe to a pressureless fluid which sets in at an epoch $t = t_i$. The density of the pressureless fluid assumes importance as matter component after the epoch t_i. In Model (ii), we consider interactions among the three fluids of the emergent universe at time $t = t_0$. Before this epoch the emergent universe can be realized without an interaction among the fluids. The problem with earlier cosmological realizations of the emergent universe was that once the EoS parameter A is fixed at a given value, the universe is unable to come out of the phase with a given composition of fluids. In the present work we overcome this problem by assigning an interaction among the fluids at the epoch t_0. A cosmological evolution of the observed universe through unified dynamics of associated matter and dark energy components thus becomes feasible in the emergent universe scenario. It is evident that a early universe with a radiation dominated phase transits to a matter dominated phase with all the features observable at the present moment with the onset of interaction considered here.

3.1.1 Thermodynamics and EU model

Generalized Second Law of Thermodynamics in an interacting fluid EU Model is explored considering the universe as a thermodynamical system. We determine the entropy and non-negativity of the time rate of change of S_{total} demonstrates the validity of the second law of thermodynamics in the context of the emergent universe model.

3.1.2 On the Origin of Initial Einstein Static Phase

Recently the origin of initial static Einstein universe is explored in massive gravity theory. A wormhole solution is found in the framework of massive gravity which permits such a static universe needed for EU initial phase which however does not permitted in GTR.

4 Cosmology II : Chaplygin Gas as Modified Matter sector

In the framework of GTR, ordinary matter fields available in the standard model of particle physics fails to account for the present observations of the universe. It is therefore essential to look for a new physics or a new type of matter that in the matter sector of the Einstein-Hilbert action. Chaplygin gas (CG) was considered to be one such candidate for dark energy. The equation of state (henceforth, EoS) for CG is

$$p = -\frac{A}{\rho} \tag{35}$$

where A is a positive constant. The initial idea of a CG originated in aerodynamics [64]. But CG is ruled out in cosmology as cosmological models are not consistent with observations [66, 67]. Subsequently, CG is generalized to incorporate different aspects of the observational universe known as generalized Chaplygin gas (in short, GCG) [71, 67] which is given by

$$p = -\frac{A}{\rho^{\alpha}} \tag{36}$$

with $0 \leq \alpha \leq 1$. It has two free parameters A and α. It is known that GCG is capable of explaining the background dynamics and various other features of a homogeneous isotropic universe satisfactorily. However, the features that the GCG corresponds to almost dust ($p = 0$) at high density does not agree completely with our universe. It is also known that the model suffers from a serious problem at the perturbative level. The matter power spectrum of GCG exhibits strong oscillations or instabilities, unless GCG model reduces to ΛCDM [73]. The oscillations for the baryonic component with GCG leads to undesirable features in CMB spectrum [74]. In order to use the gas equation in cosmology, further modification is done by adding term linear in density to the EoS which is known as modified Chaplygin gas (in short MCG). The EoS for MCG is given by:

$$p = B\rho - \frac{A}{\rho^{\alpha}} \tag{37}$$

where A, B, α are positive constants with $0 \leq \alpha \leq 1$. The MCG contains one more free parameter, namely, B, over the GCG. It may be pointed out here that the MCG is a single fluid model which unifies dark matter and dark energy. The MCG model is suitable for obtaining constant negative pressure at low density accommodating late acceleration, and a radiation dominated era (with $B = \frac{1}{3}$) at high density. Thus a universe with a MCG may be described starting from the radiation epoch to the epoch dominated by the dark energy consistently. On the other hand the GCG describes the evolution of the universe from matter dominated to a dark energy dominated regime (as $B = 0$). So compared to GCG, the proposed MCG is suitable to describe the evolution of the universe over a wide range of epoch [76].

Cosmological models are analyzed here using various observational data, namely, the redshift distortion of galaxy power spectra [77], root mean square ($r.m.s$) mass fluctuation ($\sigma_8(z)$) obtained from galaxy and Ly-α surveys at various redshifts [78, 79], weak lensing statistics [80], Baryon Acoustic Oscillations (BAO) [81], X-ray luminous galaxy clusters [82], Integrated Sachs-Wolfs (ISW) Effect ([83]-[84]). It is known that the redshift distortions are caused by velocity flow induced by gravitational potential gradient which evolves due to the growth of the universe. The gravitational growth index γ is an important parameter in the context of redshift distortion which is discussed in Ref. [85]. The cluster abundance evolution, however, strongly depends on $r.m.s$ mass fluctuations ($\sigma_8(z)$) [86], which will be used here for analysis of cosmological models.

The Hubble parameter in terms of redshift is obtained from Einstein's field equation with MCG as

$$H(z) = H_0 \left[\Omega_{b0}(1+z)^3 + (1-\Omega_{b0})[A_s + (1-A_s)(1+z)^{3(1+B)(1+\alpha)}]^{\frac{1}{1+\alpha}}\right]^{\frac{1}{2}} \tag{38}$$

where Ω_{b0}, H_0 represents the present baryon density and Hubble parameter respectively. The square of the sound speed is

$$c_s^2 = \frac{\delta p}{\delta \rho} = \frac{\dot{p}}{\dot{\rho}} \tag{39}$$

which reduces to

$$c_s^2 = B + \frac{A_s \alpha (1+B)}{\left[A_s + (1-A_s)(1+z)^{3(1+B)(1+\alpha)}\right]}. \tag{40}$$

which is always positive indicating that the perturbation is stable [75].

The growth rate of the large scale structures is derived from matter density perturbation $\delta = \frac{\delta \rho_m}{\rho_m}$ (where $\delta \rho_m$ represents the fluctuation of matter density ρ_m) in the linear regime [7, 6] is given by

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - 4\pi G_{eff} \rho_m \delta = 0. \tag{41}$$

The field equations for the background cosmology with MCG are

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_b + \rho_{mcg}), \tag{42}$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = -8\pi G \omega_{mcg} \rho_{mcg} \tag{43}$$

where ρ_b is the background energy density. The state parameter ω_{mcg} for MCG is

$$\omega_{mcg} = B - \frac{A_s(1+B)}{\left[A_s + (1-A_s)(1+z)^{3(1+B)(1+\alpha)}\right]}. \tag{44}$$

We now change t variable to $\ln a$ in eq. (41) for numerical analysis, which gives

$$(\ln \delta)'' + (\ln \delta)'^2 + (\ln \delta)' \left[\frac{1}{2} - \frac{3}{2}\omega_{mcg}(1 - \Omega_m(a))\right] = \frac{3}{2}\Omega_m(a) \tag{45}$$

where $\Omega_m(a) = \frac{\rho_m}{\rho_m + \rho_{mcg}}$. The effective matter density $\Omega_m = \Omega_b + (1-\Omega_b)(1-A_s)^{\frac{1}{1+\alpha}}$ [87]. We now change the variable from $\ln a$ to $\Omega_m(a)$ once again, the eq. (45) can be expressed in terms of the logarithmic growth factor $f = \frac{d \log \delta}{d \log a}$ which is given by

$$3\omega_{mcg}\Omega_m(1-\Omega_m)\frac{df}{d\Omega_m} + f^2 + f\left[\frac{1}{2} - \frac{3}{2}\omega_{mcg}(1 - \Omega_m(a))\right] = \frac{3}{2}\Omega_m(a). \tag{46}$$

In a flat universe, the dark energy state parameter ω_0 is a constant. For a ΛCDM model, $\gamma = \frac{6}{11}$ [85, 88], for a matter dominated model regime $\gamma = \frac{4}{7}$ [90, 91]. One can also express γ in terms of redshift parameter z. One such parametrization is $\gamma(z) = \gamma(0) + \gamma' z$, with $\gamma' \equiv \frac{d\gamma}{dz}|_{(z=0)}$ [93, 94]. It is shown recently [92] that the parametrization smoothly interpolates a low and intermediate redshift range to a high redshift range [95]. We consider the following ansatz

$$f = \Omega_m^{\gamma(\Omega_m)}(a) \tag{47}$$

where the growth index parameter $\gamma(\Omega_m)$ can be expanded in Taylor series around $\Omega_m = 1$ as

$$\gamma(\Omega_m) = \gamma|_{(\Omega_m=1)} + (\Omega_m - 1)\frac{d\gamma}{d\Omega_m}|_{(\Omega_m=1)} + O(\Omega_m - 1)^2. \tag{48}$$

Equation (48) in terms of γ becomes

$$3\omega_{mcg}\Omega_m(1-\Omega_m)\ln\Omega_m\frac{d\gamma}{d\Omega_m} - 3\omega_{mcg}\Omega_m(\gamma - \frac{1}{2}) + \Omega_m^{\gamma} - \frac{3}{2}\Omega_m^{1-\gamma} + 3\omega_{mcg}\gamma - \frac{3}{2}\omega_{mcg} + \frac{1}{2} = 0. \tag{49}$$

Differentiating once again the above equation around $\Omega_m = 1$, one obtains zeroth order term in the expansion for γ which is given by

$$\gamma = \frac{3(1-\omega_{mcg})}{5-6\omega_{mcg}}, \tag{50}$$

this is in consequence with dark energy model with a constant ω_0. In the same way differentiating the expression twice and thereafter by a Taylor expansion around $\Omega_m = 1$, one obtains a first order term in the expansion which is given by

$$\frac{d\gamma}{d\Omega_m}|_{(\Omega_m=1)} = \frac{3(1-\omega_{mcg})(1-\frac{3\omega_{mcg}}{2})}{125(1-\frac{6\omega_{mcg}}{5})^3}. \tag{51}$$

Substituting it in eq. (48), γ up to the first order term becomes

$$\gamma(B,\alpha,A_s) = \frac{3(1-\omega_{mcg})}{5-6\omega_{mcg}} + (1-\Omega_m)\frac{3(1-\omega_{mcg})(1-\frac{3\omega_{mcg}}{2})}{125(1-\frac{6\omega_{mcg}}{5})^3}. \tag{52}$$

Using the expression of ω_{mcg} in the above, γ may be parametrized in terms of B, α, A_s and z. We define normalized growth function g as

$$g(z) \equiv \frac{\delta(z)}{\delta(0)}. \tag{53}$$

The corresponding approximate normalized growth function obtained from the parametrized form of f which follows from eq. (47) is given by

$$g_{th}(z) = \exp\left[\int_1^{\frac{1}{1+z}} \Omega_m(a)^{\gamma}\frac{da}{a}\right]. \tag{54}$$

The redshift distortion parameter β, is related to the growth function f as $\beta = \frac{f}{b}$, where b represents the bias factor connecting total matter perturbation (δ) and galaxy perturbations (δ_g) ($b = \frac{\delta_g}{\delta}$) [96, 97, 99, 100]. The values for β and b at various redshifts are obtained from cosmological observations [96, 98] considering ΛCDM model. Here we analyze cosmological models in the presence of MCG using cosmic growth function. Various power spectrum amplitudes of Lyman-α forest data in SDSS are also useful to determine β.
The *Chi*-square function for growth parameter f is defined as

$$\chi_f^2(A_s,B,\alpha) = \Sigma\left[\frac{f_{obs}(z_i) - f_{th}(z_i,\gamma)}{\sigma_{f_{obs}}}\right]^2 \tag{55}$$

where f_{obs} and $\sigma_{f_{obs}}$ are observed values. However, $f_{th}(z_i, \gamma)$ is obtained from eqs. (47) and (52). Another probe for the matter density perturbation $\delta(z)$ is derived from the redshift dependence of the $r.m.s$ mass fluctuation $\sigma_8(z)$. A new Chi-square function is defined which is given by

$$\chi_s^2(A_s, B, \alpha) = \Sigma \left[\frac{s_{obs}(z_i, z_{i+1}) - s_{th}(z_i, z_{i+1})}{\sigma_{s_{obs,i}}} \right]^2 \tag{56}$$

where s_{obs}, s_{th} are observed and theoretical values respectively. From the Hubble parameter *vs.* redshift data (OHD) [101] another *Chi-square* $\chi^2_{(H-z)}$ function is defined which is given by

$$\chi^2_{(H-z)}(H_0, A_s, B, \alpha, z) = \sum \frac{[H(H_0, A_s, B, \alpha, z) - H_{obs}(z)]^2}{\sigma_z^2} \tag{57}$$

where $H_{obs}(z)$ is the observed Hubble parameter at redshift (z) and σ_z is the error associated with that particular observation. The total Chi-square function is further defined to analyze which is given by

$$\chi^2_{total}(A_s, B, \alpha) = \chi^2_f(A_s, B, \alpha) + \chi^2_s(A_s, B, \alpha) + \chi^2_{(H-z)}(A_s, B, \alpha). \tag{58}$$

The best-fit values are obtained first by minimizing the Chi-square function thereafter the contours are drawn at different confidence limit.

4.1 Results from Numerical Analysis :

The best-fit values of the EoS parameters are obtained minimizing $\chi^2_f(A_s, B, \alpha)$ making use of the growth rate data. The corresponding contours are drawn relating A_s and B. Using the best-fit values of the parameters A_s, B, α are A_s= 0.81, $B = -0.10$, α= 0.02 obtained from analysis are used to determine constraints : (i) $0.6638 < A_s < 0.8932$ and $-0.9758 < B < 0.1892$ at 95.4 % confidence limit.

Using best-fit values of the parameters A_s= 0.816, B= -0.146, α= 0.004. A_s, B, α once again in $\chi^2_f(A_s, B, \alpha) + \chi^2_s(A_s, B, \alpha)$, contours are drawn for A_s with B which puts the following constraints: (i) $0.6649 < A_s < 0.896$ and $-1.5 < B < 0.1765$ at 95.4 % confidence limit.

Finally, a total *Chi-square* function $\chi^2_{tot}(A_s$,B,$\alpha)$ is defined and minimizing it the best fit values are A_s= 0.769, B= 0.008, α= 0.002 employed to draw contours and the following limiting values (i) $0.6711 < A_s < 0.8346$ and $-0.1412 < B < 0.1502$ at 95.4 % confidence limit are noted. It is observed that at 2σ level A_s ($0.6711 < A_s < 0.8346$) admits positive values but B can take either a positive or negative value in the range ($-0.1412 < B < 0.1502$). Thus a viable cosmological model is permitted here with all the three parameters which are positive.

The growth index (γ) with redshift (z) is plotted in fig. (2). The growth index (γ) is found to lie between 0.562 to 0.60 for the redshift $z = 0$ to $z = 5$. A smooth

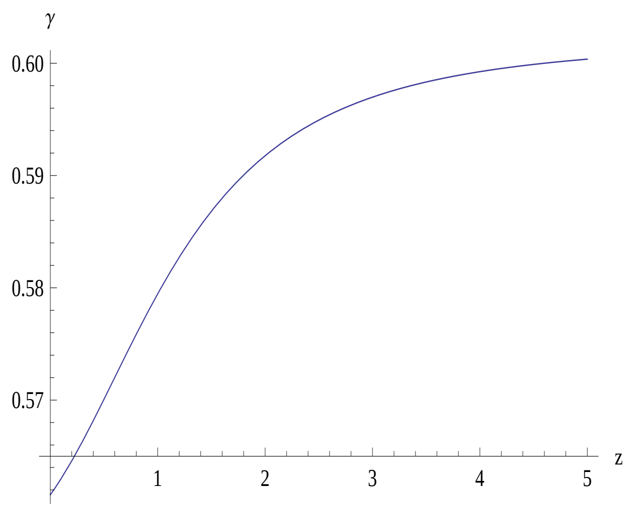

Figure 2: Evolution of growth index γ with redshift at best-fit values: A_s= 0.769, B= 0.008, α= 0.002

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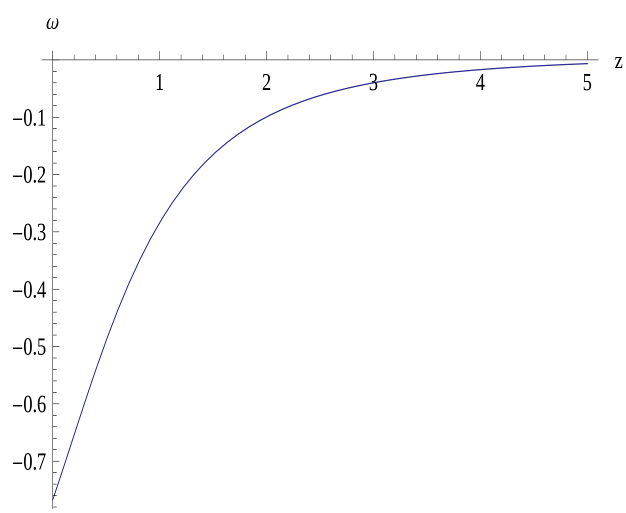

Figure 3: Evolution of the state parameter (ω) at best-fit values: A_s= 0.769, B= 0.008, α= 0.002

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fall of γ at low redshift is noticed.

The state parameter (ω) with z is plotted in fig. (3). Here, (ω) varies from -0.767 at the present epoch ($z = 0$) to $\omega \to 0$ at intermediate redshift ($z = 5$). The result is in support of the observation that present universe is now passing through an accelerating phase which is dominated by dark energy whereas in the early universe ($z > 5$) it was decelerating. It is noted that c_s^2 varies between 0.0095 to 0.0080 in the above redshift range. A small positive value indicates the growth in the structures of the universe.

4.2 Concluding Remarks on MCG Cosmologies

Cosmological models with MCG as a candidate for dark energy is used here to estimate the range of values of the EoS parameters making use of recent observed data. The growth of perturbation for large scale structure formation in this model is studied using the theory. The observed data are employed here to study the growth of matter perturbation and to determine the range of values of growth index γ as considered in Ref. [86] with MCG similar to the method adopted in Ref. [102]. The model parameters are constrained using the latest observational data from redshift distortion of galaxy power spectra and the $r.m.s$ mass fluctuation (σ_8) from Ly-α surveys. The growth data set including the Wiggle-Z survey data are employed here for the analysis.

The best-fit values of the parameters are obtained by minimizing the function $\chi^2_{tot}(A_s$,B,$\alpha)$ for background growth data. The following constraints are obtained (i) $0.6711 < A_s < 0.8346$ and $-0.1412 < B < 0.1502$ at 95.4 percent confidence limit. However, in the 2σ level we found that A_s lies between 0.6711 and 0.8346, with B in between -0.1412 and 0.1502. Thus B may be negative. The contours for A_s $vs.$ B, A_s $vs.$ α and B $vs.$ α are drawn for growth data, growth+ σ_8 data and growth+ σ_8 + H $vs.$ z data. The best-fit value of the growth parameter at present epoch ($z = 0$) is f= 0.472 with growth index γ= 0.562, state parameter ω=-0.767 and Ω_{m0}= 0.262, which are in good agreement with the ΛCDM model. It is also noted that the growth function f varies between 0.472 to 1.0 and the growth index γ varies between 0.562 to 0.60 for a variation of redshift from $z = 0$ to $z = 5$. In this case the state parameter ω lies between -0.767 to 0, square of the sound speed is $c_s^2 < 1$ always.

Here the growth and Hubble data are employed to test the suitability of MCG in FRW universe. It is found that a satisfactory cosmological model emerges permitting present accelerating universe. The negative values of state parameter ($\omega \leq -\frac{1}{3}$) signifies the existence of such a phase of the universe. Thus it is noted that MCG is a good candidate for a universe which can reproduce the cosmic growth with inhomogeneity admitting a late time accelerating phase. The observational constraints that are estimated in the MCG model are agreeing close to ΛCDM model compared to the GCG model. It may be mentioned here that the MCG model reduces to GCG model for $B = 0$ and ΛCDM model for $B = 0$ and $\alpha = 0$. In Table-(1) a comparison of values of EoS parameters corresponding to previously probed GCG model with that of ΛCDM, GCG and MCG models obtained by us are also shown.

Model	Data	A_s	α	B	$Ref.$
GCG	$Supernovae$	0.6-0.85	–	0	[68]
GCG	$CMBR$	0.81-0.85	0.2-0.6	0	[69]
GCG	$WMAP$	0.78-0.87	–	0	[67]
GCG	$CMBR + BAO$	≈ 0.77	≤ 0.1	0	[70]
GCG	$Growth + \sigma_8 + OHD$	0.708	-0.140	0	this paper
MCG	$Growth + \sigma_8 + OHD$	0.769	0.002	0.008	this paper
ΛCDM	$Growth + \sigma_8 + OHD$	0.761	0	0	this paper

Table 1: Comparison of the values of EoS parameters for ΛCDM, GCG and MCG models

5 Relativistic Astrophysics

5.1 Relativistic Solutions of Compact Objects and Stellar Models

In the last couple of decades precision astronomical observations predicted existence of massive compact objects with very high densities [104]. The theoretical investigation of such compact astrophysical objects has been a key issue in relativistic astrophysics since then. In general a white dwarf can be desribed by a polytropic equation of state (EOS) which is a less compact star [105]. However, theoretical understanding in the last couple of decades made it clear that there is a deviation from local isotropy of the pressure inside compact objects of high enough densities with smaller radial size [106, 107]. The physical situations where anisotropic pressure may be relevant are very diverse for a compact stellar object [106, 108, 109, 107]. By anisotropic pressure we mean the radial component of the pressure (p_r) different from that of the tangential pressure p_t. After the seminal work of Bowers and Liang [109], Ruderman and Canuto [106, 107] theoretically investigated compact objects and observed that a star with matter density ($\rho > 10^{15} gm/cc$), where the nuclear interaction become relativistic in nature, are likely to be anisotropic. It is further noted that anisotropy in fluid pressure in a star may originate due to number of processes e.g., the existence of a solid core, the presence of type 3A super fluid etc. [110]. Recently, Mak and Harko [111] determined the maximum mass and mass to radius ratio of a compact isotropic relativistic star. Bowers and Liang[109], Bayin[112], Maharaj and Maartens [108] examined spherical distribution of anisotropic matter in the framework of general relativity and derived a number of solutions to understand the interior of such stars. A handful number of exact interior solutions in general relativity for both the isotropic and the anisotropic compact objects are reported in the literature [113]. Delgaty and Lake [113] analysed 127 published solutions out of which they found that only 16 of the published results satisfy all the conditions for a physically viable stellar model. In the case of a compact stellar object it is essential to satisfy all the conditions outlined by Delgaty and Lake as the EOS of the fluid of the compact dense object is not known.

Discovery of a handful of compact objects, namely, Her X1, millisecond pulsar SAX J1808.43658, X-ray sources, 4U 1820-30 and 4U 1728-34 led to a belief that these may belong to strange star class. The existence of such characteristics compact objects led to critical studies of stellar configurations [114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126]. The EoS of matter inside a superdense strange star at present is not known. Vaidya-Tikekar [105] and Tikekar [122] have shown that in the absence of definite information about the EoS, an alternative approach of prescribing a suitable *ansatz* for the geometry of the interior physical 3-space of the configuration leads to simple easily accessible physically viable models of such stars. Relativistic models of superdense stars based on different solutions of Einstein's field equations obtained by Vaidya-Tikekar approach of assigning different geometries with physical 3-spaces of such objects are reported in the literature [116, 118, 121, 124, 125]. Pant and Sah [127] obtained a class of relativistic static non-singular analytic solutions in isotropic form with a spherically symmetric distribution of matter in a static space time. Pant and Sah solution is found to lead to a physically viable causal model of neutron star with a maximum mass of $4M_{\odot}$. Recently, Deb *et. al.* [128] obtained a class of compact stellar models using Pant and Sah solution in the case of spherically symmetric space time. Further we obtain a class of new relativistic solution which accommodates compact objects with anisotropic pressure having mass relevant for a neutron star.

For a static spherically symmetric space time metric given by

$$ds^2 = e^{\nu(r)}dr^2 - e^{\mu(r)}(dr^2 + r^2 d\Omega^2) \tag{59}$$

where $\nu(r)$ and $\mu(r)$ are unknown metric functions and $d\Omega^2 = d\theta^2 + sin^2\theta \; d\phi^2$. The field equations are

$$-e^{-\mu}\left(\mu'' + \frac{\mu'^2}{4} + \frac{2\mu'}{r}\right) = \rho, \tag{60}$$

$$e^{-\mu}\left(\frac{\mu'^2}{4} + \frac{\mu'}{r} + \frac{\mu'\nu'}{2} + \frac{\nu'}{r}\right) = p_r, \tag{61}$$

$$e^{-\mu}\left(\frac{\mu''}{2} + \frac{\nu''}{2} + \frac{\nu'^2}{4} + \frac{\mu'}{2r} + \frac{\nu'}{2r}\right) = p_t. \tag{62}$$

Anisotropy is defined as

$$\left(\frac{\mu''}{2} + \frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\mu'^2}{4} - \frac{\mu'}{2r} - \frac{\nu'}{2r} - \frac{\mu'\nu'}{2}\right) = \Delta e^{\mu}. \tag{63}$$

The above equation is a second-order differential equation which admits a class of new solution with anisotropic fluid distribution given by

$$e^{\frac{\nu}{2}} = A\left(\frac{1 - k\alpha + n\frac{r^2}{R^2}}{1 + k\alpha}\right), \qquad e^{\frac{\mu}{2}} = \frac{(1 + k\alpha)^2}{1 + \frac{r^2}{R^2}} \tag{64}$$

where

$$\alpha(r) = \sqrt{\frac{1 + \frac{r^2}{R^2}}{1 + \lambda\frac{r^2}{R^2}}} \tag{65}$$

with R, λ, k, A and n are arbitrary constants. The above constants are to be determined using the physical conditions imposed on the solutions for a consistent stellar model. For $n = 0$ recovers the relativistic solution obtained by Pant and Sah [127] for isotropic fluid distribution. The non-zero value of n here corresponds to anisotropic star.

The geometry of the 3-space is given by

$$d\sigma^2 = \frac{dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2)}{1 + \frac{r^2}{R^2}}. \tag{66}$$

It corresponds to a 3 sphere immersed in a 4-dimensional Euclidean space. Accordingly the geometry of physical space obtained at the $t = constant$ section of the space time is given by

$$ds^2 = A^2 \left(\frac{1 - k\alpha + n\frac{r^2}{R^2}}{1 + k\alpha} \right)^2 dt^2 - \frac{(1 + k\alpha)^4}{(1 + \frac{r^2}{R^2})^2} \left[dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2) \right]. \tag{67}$$

For $\lambda > 0$, the solution corresponds to finite boundary. The physical properties of compact objects filled with anisotropic fluid ($n \neq 0$) are then studied with different R, λ, k and A for a viable stellar model. The solutions is also matched with the exterior Schwarzschild line element

$$ds^2 = \left(1 - \frac{2m}{r_o}\right) dt^2 - \left(1 - \frac{2m}{r_o}\right)^{-1} dr^2 - r_o^2(d\theta^2 + sin^2\theta d\phi^2) \tag{68}$$

at the boundary, where m represents the mass of spherical object. The isotropic form of the metric [5] is

$$ds^2 = \left(\frac{1 - \frac{m}{2r}}{1 + \frac{m}{2r}} \right)^2 dt^2 - \left(1 + \frac{m}{2r}\right)^4 (dr^2 + r^2 d\Omega^2) \tag{69}$$

using $r_o = r\left(1 + \frac{m}{2r}\right)^2$ where r_o is the radius of the compact object [5].

5.1.1 Physical properties of anisotropic compact star

The steps for the analysis is outlined below:

(1) In this model, a positive central density ρ is obtained for $\lambda < \frac{4}{k} + 1$.

(2) At the boundary of the star ($r = b$), the interior solution should be matched with the isotropic form of Schwarzschild exterior solution, i.e.,

$$e^{\frac{\nu}{2}}|_{r=b} = \left(\frac{1 - \frac{m}{2b}}{1 + \frac{m}{2b}} \right) \qquad e^{\frac{\mu}{2}}|_{r=b} = \left(1 + \frac{m}{2b}\right)^2 \tag{70}$$

(3) The physical radius of a star (r_o), is determined knowing the radial distance where the pressure at the boundary vanishes (i.e., $p(r) = 0$ at $r = b$). The physical radius is related to the radial distance ($r = b$) through the relation $r_o = b\left(1 + \frac{m}{2b}\right)^2$ [5].

(4) The ratio $\frac{m}{b}$ is determined using eqs. (8) and (14), which is given by

$$\frac{m}{b} = 2 \pm 2A \left(\frac{1 - k\alpha + ny^2}{\sqrt{1 + y^2}} \right). \tag{71}$$

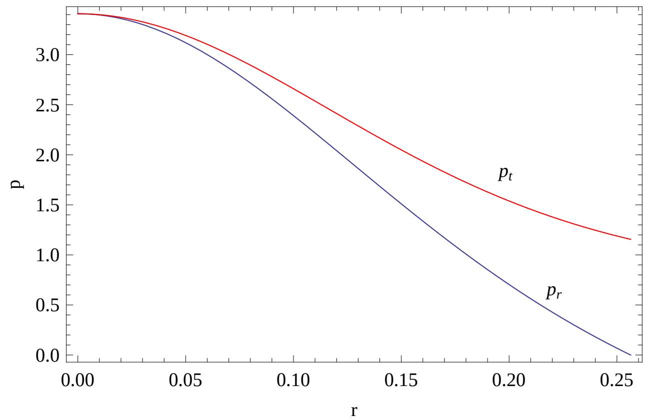

Figure 4: Radial variation of transverse and radial pressure with $\lambda = 10$, $n = 0.8$, $A = 2$ and $k = 0.31$. Blue line for radial pressure and red line for transverse pressure.

In the above we consider only negative sign as it corresponds to a physically viable stellar model.

(5) The density inside the star should be positive i.e., $\rho > 0$.

(6) Inside the star the stellar model should satisfy the condition, $\frac{dp}{d\rho} < 1$ for the sound propagation to be causal.

The usual boundary conditions are that the first and second fundamental forms required to be continuous across the boundary $r = b$. We determine n, k, λ and A which satisfy the criteria for a viable stellar model outlined above. As the field equations are highly non-linear and intractable to obtain a known functional relation between pressure and density we adopt numerical technique. Imposing the condition that the pressure at the boundary vanishes, we determine y which is given by,

$$y = \frac{b}{R} \tag{72}$$

using eq. (5). The square of the acoustic speed $\frac{dp}{d\rho}$ becomes :

$$\frac{dp}{d\rho} = -\frac{\sqrt{\alpha}(1 + k\sqrt{\alpha})(A + \frac{B}{\sqrt{\alpha}} + C + D)}{E} \tag{73}$$

where A, B, C, D are functions of r, k, n, λ For given values of λ and k, the size of the star is estimated. The mass to radius $\frac{m}{b}$ of a star is then determined, which in turn determines the physical size of the compact star (r_o).

The radial variation of pressure and the density of anisotropic compact objects for different parameters are studied. It is found that radial pressure increases with an increase in k whereas the density decreases. The central density increases with a decrease in k. The pressure inside the star decreases if n is increased, however, density is independent. The density and pressure are found to increase with an increase in λ value showing an increase in corresponding central density but the difference between central density with surface density decreases with λ. We also observed that pressure and density do not depend on A. The decrease in radial pressure near the boundary falls rapidly for higher values of λ. The variation of radial and transverse pressure

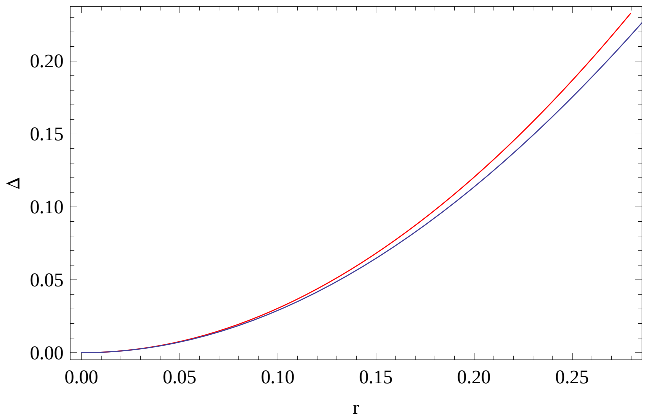

Figure 5: Radial variations of anisotropic parameter Δ for different n. Blue line for $n = 0.7$ and red line for $n = 1$.

are plotted in fig. (?), the magnitude of transverse pressure is more than that of radial pressure although they begin with same central pressure. We note that there exist a region near the center of the star where strong energy condition (SEC) is not obeyed. The radius of that region is found to increase with an increase in the parameter values of n, k and λ. Thus the relativistic solution obtained here may be useful for constructing a core-envelope model of the star. The radial variation of anisotropy inside the star for different n values are plotted in fig. (13) which shows that a higher anisotropic star corresponds to a higher values of n.

5.1.2 Stellar Models

For a given mass of a compact star, it is possible to estimate the corresponding radius in terms of the geometric parameter R. To obtain stellar models we consider compact objects with observed mass [130] which determines the radius of the star for different values of R with given set of values of n, A, k and λ. It is known that the radius of a neutron star should be ≤ 10 km., stellar models are obtained here taking different R so that the size of the star satisfies the upper bound. In the next section we consider observed masses of compact objects to obtain stellar models. We obtain three different models using stellar mass data [114, 115, 130] :

Model 1 : For X-ray pulsar Her X-1 [130, 131, 132] characterized by mass $m = 1.47\ M_{\odot}$, where $M_{\odot}$ = the solar mass and found that it permits a star with radius $r_o = 8.31106$ km., for $R = 8.169$ km. The compactness of the star is $u = \frac{m}{r_o} = 0.30$. The ratio of density at the boundary to that at the center for the star is 0.128 with the set of parameters $\lambda = 1.9999$, $k = 0.641$, $A = 2$ and $n = 0.697$. However, for the same mass of a compact object, stellar models with different size and compactness factor may be obtained which are tabulated in Table- 3. It is found that as the compactness increases size of the star decreases. It is evident from Table-4 that if we increase λ value which is related to geometry, the density profile is found to decrease.

Model 2 : For X-ray pulsar J1518+4904 [130, 131, 132] characterized by mass $m = 0.72\ M_{\odot}$, permits a star with radius $r_o = 4.071$ km., for $R = 8.169$ km.The

$\frac{m}{b}$	R in km.	Radius (r_o in km.)
0.3	8.169	8.311
0.28	8.574	8.828
0.26	9.048	9.424
0.25	9.317	9.757
0.20	11.096	11.925

Table 2: Variation of size of a star with $\frac{m}{b}$ for $k = 0.641$, $n = 0.697$, $\lambda = 1.9999$ and $A = 2$.

λ	1	1.1	1.2	1.3	1.4	1.5	1.7	1.9999
$\frac{\rho(b)}{\rho(0)}$	0.449	0.447	0.444	0.440	0.436	0.432	0.423	0.409

Table 3: Density profile for $k = 0.641$, $n = 0.697$ and $A = 2$.

compactness of the star in this case is $u = \frac{m}{r_o} = 0.30$. The ratio of density at the boundary to that at the centre for the star is 0.142 with $\lambda = 1.1$, $k = 0.641$, $A = 2$ and $n = 0.60$.

Model 3 : For B1855+09(g)[130, 131, 132] characterized by mass $m = 1.6\ M_{\odot}$, permits a star with radius $r_o = 9.047$ km., for $R = 8.169$ km. The compactness of the star in this case is $u = \frac{m}{r_o} = 0.30$. The ratio of density at the boundary to that at the centre for the star is 0.187 with $\lambda = 1$, $k = 0.52$, $A = 2$ and $n = 0.50$.

6 Concluding Remarks on Research in Astrophysics

Relativistic solutions of compact objects are obtained using Einstein's general theory of relativity and explored the possibility of describing features of very compact objects. In this case primordial black holes are also studied in a modified gravity framework.

6.1 Anisotropic Compact Star

A class of new general relativistic solutions for compact stars which are in hydrostatic equilibrium is obtained assuming anisotropic interior fluid distribution. The radial pressure and the tangential pressure are considered different in this case. As the EOS of the fluid inside a neutron star is not known so we employ numerical technique to obtain a probable EOS of the matter for a given space-time geometry. The interior space-time geometry considered here is characterized by five geometrical parameters namely, λ, R, k, A and n which will be used to obtain stellar models. For $n = 0$, the relativistic solution reduces to that considered in Refs. [127, 128] to construct stellar models. The permitted range of the values of the unknown parameters are determined from the following conditions : (a) metric matching at the boundary, (b) $\frac{dp}{d\rho} < 1$, (c) pressure at the boundary becomes zero and (d) the positivity of density.

We note the following: (i) The pressure is found to increase with an increase in k but the density decreases. The central density increases with an decrease in k. (ii) The presssure decreases as n increases, while the density remains unchanged. (iii) The presssure and density both increases with an increase in λ and the central density is thus increases with an increase in λ. It is further noted that the decrease in radial pressure near the boundary dips sharply for higher values of λ. (iv) A region near the center of the star exists where SEC is violated and the radius of that region increases with an increase in the value of one parameter from n, k and λ while keeping the other constant. It is useful to construct a core-envelope model of a compact star which will be studied in future. (vi) A star with large anisotropy corresponds to a large value of n. (vii) For a given λ as we increase n the reduced size of a star increases. However for $n = 0$ the size of the star increases with an increase in λ but for non-zero values of n the size of the star decrease. For a given λ the size of the star increases as k increases, but for a given k the size of the star decreases as λ increases. (viii) Considering observed masses of the compact objects namely, HER X-1, J1518+4904 and B1855+09(g) we analyze the interior of the star. A class of compact stellar models with anisotropic pressure distribution is permitted with the new solution. The models permits stars with various compactification factor which are shown in Tables-(1). The density profile inside the star is found to decrease as one increases λ. A functional relation of radial pressure with density is presented in Table-(4), this is first we predict the EoS from relativistic consideration. It is also noted that the stellar models obtained here admits neutron stars with mass less than $2M_\odot$ for an anisotropic fluid distribution which is the upper limit of mass of the star observed so far.

Given Star	Mass(m)	Radial pressure
HER X-1	$1.47M_\odot$	(i) $p_r = 1.207\rho - 8.477$ (ii) $p_r = 0.130\rho^2 - 1.032\rho + 0.980$
J1518+4904	$0.72M_\odot$	(i) $p_r = 1.041\rho - 7.607$ (ii) $p_r = 0.104\rho^2 - 0.794\rho + 0.350$
B1855+09(g)	$1.6M_\odot$	(i) $p_r = 0.602\rho - 5.316$ (ii) $p_r = 0.043\rho^2 - 0.252\rho - 1.151$

Table 4: Predicted EoS for different stellar models.

6.2 Strange Stars Model

In the case of a strange star when matter is described by MIT bag model, it is found that the Bag parameter in a strange star may not be a constant and we determine the functional relation of such bag parameter as a function of density.

A framework is developed to study the effects of pressure anisotropy on the evolution of a collapsing star dissipating energy in the form of radial heat flux. A static star configuration described by Paul and Deb (Astrophys. Space Sci. 354:421, 2014) solution is probed in the presence (or absence) of anisotropy. A compact star begins its collapse from an initial static phase, in this case we develop a simple method to analyze the effects of anisotropy onto the collapse.

6.3 PBH in $f(T)$- Gravity

Black holes are considered as the most elusive object in Astrophysics and cosmology. It is known in stellar physics that Black holes are the ultimate corpse of a collapsing massive star when its mass exceeds twice the mass of the sun. Another kind of black holes are important in cosmology which might have formed in the early universe during inflation [133, 134], initial inhomogeneities [135, 136] , phase transition and critical phenomena in gravitational collapse [137, 138, 139, 140, 141, 142, 143]. The topological black holes are formed during inflation due to quantum fluctuations and they possess mass much smaller than a solar mass. The later type of black holes are popularly known as primordial black holes (in short, PBH). The PBH are important from the Hawking radiation perspective as they are hotter than the cosmic background. PBH might play a very important role in contributing total density of the universe. The work of Hawking [145] reflected that the black holes emit thermal radiation due to quantum effects which might evaporate, the time scale of evaporation depends on their masses. It may be that the PBH mass could be small enough for them to be evaporated completely by the present epoch due to Hawking radiation. Early evaporating PBHs might have contributed for baryogenesis [146, 147, 148]. On the otherhand longer lived PBHs could act as seed for the structure formation or generating supermassive black hole [149, 150]. It is also possible for the PBHs to survive even longer to date and can contribute a significant component of dark matter [151, 152].

The effect of vacuum energy on the evolution of PBH in GTR is investigated [153] and found that the vacuum energy does not play any role on the evaporating time scale of PBH. The time scale of accretion and the evolution of mass of black holes are studied in the presence of modified variable Chaplygin gas and viscous generalized Chaplygin gas [154]. In an anisotropic Bianchi-I universe accretion of PBH during radiation dominated universe is studied [156] and found that the life time is longer than isotropic universe. Dark matter and density perturbation are studied [157] in the presence of PBH. [158] studied the large scale structure formation from PBH evaporation. The effect of PBH on 21 cm fluctuation is studied recently [159]. PBHs are also considered as a tool for constraining non-gaussianity [160, 161] in the large scale structure formations. For understanding further we study PBH in the $f(T)$-gravity, a modified theory of gravity. We studied the accretion of PBH in $f(T)$ gravity in the presence of modified Chaplygin gas. We note that initially accretion dominated, the rate of increase of accretion over evaporation upto a certain epoch is dominating. However at late time, the accretion increases sharply attains a maximum thereafter decreases. The decrease of mass is due to the fact that the evaporation rate is more than that of accretion. The mass evaporated from a PBH may contribute to the dark energy of the universe. A strange situation is noted at $z = -1$ when suddenly rate of evaporation of primordial black holes becomes large.

7 List of Published Papers : Cosmology & Astrophysics

1. Paul B C & Thakur P, *Observational Constraints on Modified Chaplygin Gas from Cosmic Growth* **JCAP 11** 052 (2013)

2. Paul B C, *Dark Matter and Dark Energy in the Universe* **Bibechana 11** 421-430 (2014)

3. Panigrahi D, Paul B C & Chatterjee S, *Constraining Modified Chaplygin Gas Parameters* **Gravitation and Cosmology 356** 327-337 (2015)

4. Paul B C & Majumdar A S, *Emergent Universe with interacting fluids and Generalized Second law of Thermodynamics* **Class. Quantum Grav. 32** 115001 (2015)

5. Paul B C & Thakur P, *Observational Constraints on Chaplygin Gas: A Review* Chapter-7 in An Introduction to Astronomical Data Amnalysis ed: B C Paul 149-177 (2015).

6. Paul Bikash Chandra & Deb Rumi, *Relativistic solutions of Anisotropic Compact Objects* **Astrophys. & Space Sci. 354** 421-430 (2014)

7. Chattopadhyay P K, Deb R & Paul B C , *Relativistic charged solutions in Higher Deimensions* **Int. J. Theor. Phys. 53** 1666-1684 (2014)

8. Paul B C, Chattopadhyay P K & Karmakar S, *Relativistic anisotropic star and its maximum mass in Higher Deimensions* **Astrophys. & Space Sci. 356** 327-337 (2015)

9. Chattopadhyay P K & Paul B C , *Density dependent B parameter of Relativistic stars with anisotropy in Pseudo-spheroidal spacetime* **Astrophys. Space Sci. 361** 145 (2016)

10. Debnath U & Paul B C, *Evolution of Primordial Black hole in $f(T)$-gravity with modified Chaplygin gas* **Astrophys. Space Sci 355** 147-153 (2015)

11. Das S, Sharma R, Paul B C & Deb R, *Dissipative Gravitational Collapse of an anisotropic star* **Astrophys. Space Sci. 361** 1 (2016).

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